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Antony Millner
Simon Dietz
Geoffrey Heal

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ABSTRACT

Economic evaluation of climate policy traditionally treats uncertainty by appealing to expected utility theory. Yet our knowledge of the impacts of climate policy may not be sufficiently high quality to justify probabilistic beliefs. In such circumstances, it has been argued that the axioms of expected utility theory may not be the correct standard of rationality. Several axiomatic frameworks have recently been proposed to account for ambiguous beliefs. We apply static and dynamic versions of the smooth ambiguity model of Klibanoff et al, (2005, 2009) to climate policy. We illustrate via comparative statics the conditions under which an increase in ambiguity aversion increases the optimal level of mitigation in some simple examples. We then extend our analysis to a dynamic setting and adapt the well-known DICE model of the climate-economy system to show that the value of emissions abatement increases as ambiguity aversion increases. We also find that the value of abatement is more sensitive to risk aversion than to ambiguity aversion because, according to our data, the inter-model spread in average consumption growth is small relative to its mean value. However this is an empirical matter and we show that under some conditions ambiguity aversion can have a significant effect on the value of abatement.

Antony Millner
London School of Economics
Houghton Street
Aldwych, London WC2A 2AE
England
A.Millner@lse.ac.uk

Simon Dietz
London School of Economics
Houghton Street
Aldwych London WC2A 2AE
England
S.Dietz@lse.ac.uk

Geoffrey Heal
Graduate School of Business
616 Uris Hall
Columbia University
New York, NY 10027-6902
and NBER
gmh1@columbia.edu

Ambiguity and climate policy

Antony Millner*[1], Simon Dietz[1,2], and Geoffrey Heal[3,4]

[1]Grantham Research Institute on Climate Change and the Environment, London School of Economics and Political Science

[2]Department of Geography and Environment, London School of Economics and Political Science

[3]Columbia Business School, Columbia University

[4]National Bureau of Economic Research

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Abstract

Economic evaluation of climate policy traditionally treats uncertainty by appealing to expected utility theory. Yet our knowledge of the impacts of climate policy may not be sufficiently high quality to justify probabilistic beliefs. In such circumstances, it has been argued that the axioms of expected utility theory may not be the correct standard of rationality. By contrast, several axiomatic frameworks have recently been proposed that account for ambiguous beliefs. In this paper, we apply static and dynamic versions of the smooth ambiguity model of Klibanoff et al. (2005, 2009) to climate mitigation policy. We illustrate via comparative statics the conditions under which an increase in ambiguity aversion increases the optimal level of mitigation in some simple examples. We then extend our analysis to a more realistic, dynamic setting, and adapt a well-known empirical model of the climate-economy system to show that the value of emissions abatement increases as ambiguity aversion increases. We also find that the value of abatement is more sensitive to risk aversion than it is to ambiguity aversion for the simple reason that, according to our data, the inter-model spread in average consumption growth is small relative to its mean value. However, this is an empirical question, and we show that under certain conditions ambiguity aversion can have a significant effect on the value of abatement.

1 Introduction

The literature on optimal climate change mitigation policy has thus far remained faithful to the long tradition of welfare analysis based on expected utility maximization. The integrated assessment models that are widely used for policy evaluation all have a common welfare-analytic core. Most studies employ deterministic models (Manne & Richels, 1992; Nordhaus, 2008; Tol, 1997), allowing for efficient determination of optimal policies, which are then subjected to sensitivity analysis in order to test their robustness to changes in model parameters. Other studies employ

*Email address for correspondence: A.Millner@lse.ac.uk

stochastic models (Hope, 2006), and generally do not find optimal policies, but rather provide welfare assessments of exogenously specified greenhouse gas emissions pathways. A few authors have combined these two approaches by solving stochastic-dynamic control problems to endogenously determine optimal policies that account for future risks (Pizer, 1999; Keller et al., 2004; Kelly & Kolstad, 1999). Thus, while there are several models of varying complexity and emphasis, they share a common commitment to the expected utility framework.

The reasons for the primacy of expected utility theory as a normative model of rational choice are well known to economists. Its axiomatic foundations have been developed by several authors (von Neumann & Morgenstern, 1944; Savage, 1954; Anscombe & Aumann, 1963). Savage's presentation is widely considered the most satisfactory, since it derives both utility functions and subjective probabilities from primitive preferences over 'acts', i.e. maps between states and outcomes. Indeed his axioms are often considered to be synonymous with rational choice. Nevertheless, Savage himself took a cautious approach to his theory, suggesting that it should only be applied in small worlds, in which it is possible to 'look before you leap', i.e. imagine every possible contingency, and identify a complete ordering of acts over these contingencies (see Binmore, 2009, for a discussion). Further potential limitations on the domain of applicability of Savage's theory were famously identified by Ellsberg (1961), who showed that when our state of knowledge is more accurately described as uncertainty rather than risk (we use these terms in the sense of Knight (1921)), we may wish to violate Savage's second axiom, the 'sure-thing principle'. A strong Bayesian would interpret Ellsberg's results as a contribution to positive, rather than normative, decision theory. Bayesians believe that Savage's axioms define rational choice, in which case the preferences Ellsberg observes in his investigations are deemed irrational, and all uncertainty is always describable by a unique probability distribution. This viewpoint has however been strongly contested. As noted by Ellsberg (1961) and Slovic & Tversky (1974), people often stick to choices that violate the sure-thing principle in Ellsberg's choice experiments, even when this violation is pointed out to them. This is in stark contrast to other decision theoretic 'paradoxes', such as the Allais paradox, where people often revert to the prescriptions of expected utility once their violation of the axioms is explained. It has been argued, we think convincingly, that when our information about the world is incomplete, inconsistent, or nonexistent, Savage's axioms need not be the correct standard of rationality (Gilboa et al., 2008, 2009), and it does not necessarily make sense to describe our state of knowledge with a unique probability distribution over states of the world[1].

Given these views, our assessment of the validity of the expected utility approach to the welfare analysis of climate change policy must depend on how structured our beliefs about the climate system[2] are. If we have sufficiently high quality information to justify probabilistic beliefs, then the approach adopted thus far in the literature is unequivocally useful. If not, we need to justify why this approach is a useful approximation, or attempt to define welfare measures that are true to our actual state of knowledge about the climate system, and reflect our preferences over bets with unknown probabilities. Very often a good way of justifying an approximation is to embed

[1]Ellsberg himself emphasizes that '...either the postulates failed to be acceptable in those circumstances as normative rules, or they failed to predict reflective choices...But from either point of view, it would follow that *there would be simply no way to infer meaningful probabilities for those events from their choices*, and theories which purported to describe their uncertainty in terms of probabilities would be quite inapplicable in that area.'

[2]Our work focusses on uncertainty about a key parameter of the climate system – the climate sensitivity – and for the most part assumes that economic parameters are known. More on this later.

it in a more general framework, and show that the increased power of this framework does not materially alter the results achieved by the approximation. Thus, provided we suspect that our knowledge of the climate system is not very high quality, there would seem to be good reason to develop approaches to policy evaluation which account for uncertainty and not just risk, since these will either justify our reliance on existing methods, or provide appropriate tools for future work.

What is our state of knowledge about the climate system, and can it be described by unique probabilities? We feel it is important to break the state of scientific knowledge about climate into two categories: broad scientific principles, and detailed empirical predictions. In the first category belong concepts such as the laws of thermodynamics, fluid dynamics, and statements of fact such as 'CO_2 traps outgoing long-wave radiation, causing warming'. We believe these principles to be unimpeachable. In the second category belong the sophisticated models scientists use to convert these principles into predictions – energy balance models (EBMs), earth systems models of intermediate complexity (EMICs), and full-scale general circulation models (GCMs). These models can be enormously complex, and attempt to predict, among other things, the response of the global climate to increases in the concentrations of greenhouse gases. Because of the complexity of their task, and the intrinsic difficulties of prediction in highly nonlinear multi-dimensional physical systems (see Smith (2002, 2007); Stainforth et al. (2007); Frame et al. (2007) for illuminating discussions of the scientific and philosophical challenges of climate prediction), these models are not always in agreement with one another. As an example of this, consider Figure 1, which plots the results of several recent studies' attempts to use different models and observational data to estimate the climate sensitivity[3], a critical parameter for estimating the response of the climate to increases in CO_2 concentrations which features prominently in integrated assessment models. From the figure it is clear that there are many inconsistent estimates of this important quantity. This suggests that we are indeed in an environment characterized by uncertainty, rather than risk[4]. How then should policy reflect this uncertainty?

Since Ellsberg's work there has been a series of theoretical advances in decision theory which provide axiomatic representations of preferences that account for uncertainty or ambiguity. Seminal contributions include Arrow & Hurwicz (1977); Schmeidler (1989); Gilboa & Schmeidler (1989) and Klibanoff et al. (2005). These are elegant models, and have found application in several areas of economics, especially finance (e.g. Dow & da Costa Werlang, 1992; Bassett et al., 2004; Gollier, 2009; Bossaerts et al., 2010). Hansen & Sargent (2007) have applied similar techniques in macroeconomics in order to derive policies that are robust to model misspecification. Despite the appeal of these models in other areas of applied economics, they have only just begun to filter into the argument around climate policy. Henry & Henry (2002) is perhaps the first paper to view climate policy through this lens, and focusses on formalizing precautionary policies as policies that account for ambiguity in scientific knowledge. Lange & Treich (2008) provide some comparative

[3]The climate sensitivity is the amount by which global mean surface temperature rises for a doubling of CO_2 concentration, in equilibrium.

[4]There are two standard responses to this assertion: Why not simply aggregate the different estimates into a single probability density (Bayesian response); and surely all the estimates are not equally valid, so why not simply choose the 'best' estimate (scientists' response)? The point we are making is that the spread in the set of distributions is an indication of the intrinsic ambiguity in our state of knowledge. If decision makers perceive this ambiguity, and are averse to it as Ellsberg's results suggest they will be, their beliefs *cannot* be described by a single probability distribution.

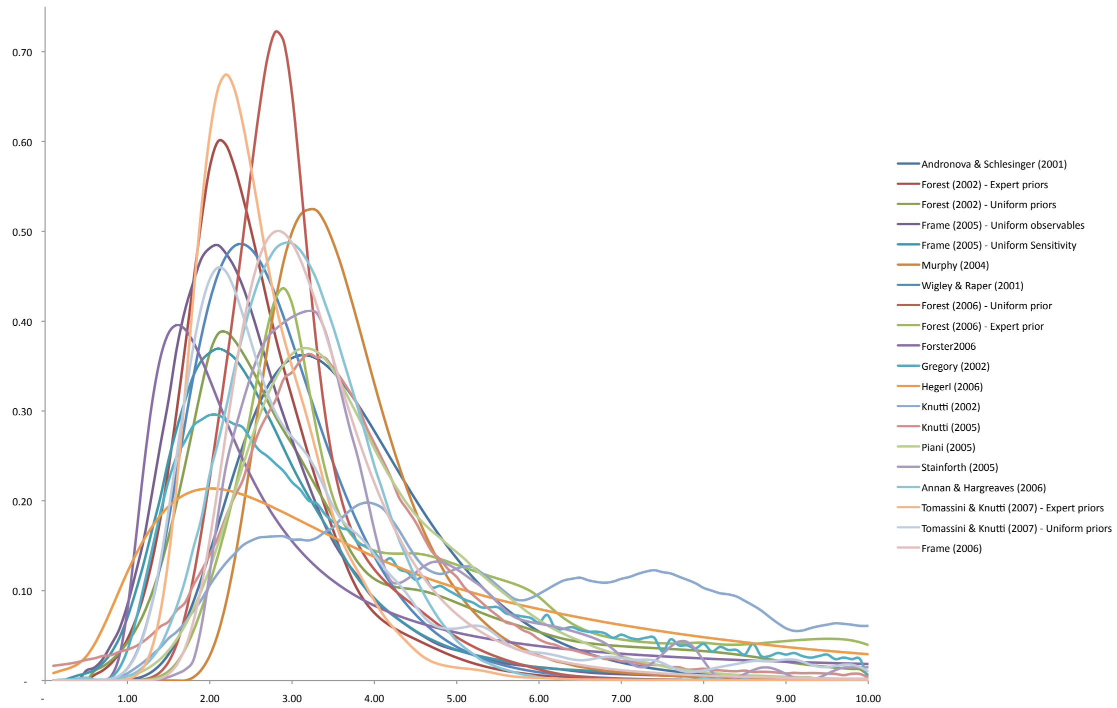

Figure 1: Estimated probability density functions for the climate sensitivity from a variety of published studies, collated by Meinshausen et al. (2009).

statics results on the effect of ambiguity on optimal abatement in a two-period model. Although not confined to climate applications, the work of Traeger (2009) and Gollier & Gierlinger (2008) on the effect of ambiguity aversion on the social discount rate is clearly relevant and has important implications for the assessment of mitigation investments.

In this paper we hope to provide a further step along the path sketched out by these authors. Our aims are two-fold: First, to provide further insight into the comparative statics of ambiguity aversion in stylized models of abatement policy choice, and second to attempt to understand how ambiguity aversion affects the welfare assessment of dynamic abatement policies. The dynamic aspects of ambiguity models are especially difficult, and we only consider their implications to a limited extent. In particular, we do not account for the dynamic resolution of ambiguity over time through learning, but rather focus on computing welfare measures in two extreme cases – when ambiguity is expected to resolve completely after one time period, and when ambiguity persists unchanged for all time. These extremes allow us to bound the welfare effects of alternative abatement policies. We compute welfare measures for sample exogenous abatement pathways which are fed into the integrated assessment model DICE (Nordhaus, 2008) to generate consumption streams, and assess how ambiguity over the correct probability density for the climate sensitivity affects them.

2 The smooth ambiguity model and optimal abatement

A potential difficulty with several of the decision models that account for ambiguity is that they do not achieve a separation between ambiguous beliefs and attitudes towards ambiguity. This was overcome by the contribution of Klibanoff et al. (2005), who provided a preference representation that separates tastes from beliefs, and allows us to parameterize attitudes to ambiguity via a differentiable function, in a manner analogous to the way utility functions represent risk preferences. Their formalism is thus perfectly suited to understanding how different degrees of ambiguity aversion affect policies and welfare estimates. We introduce their model below.

Klibanoff et al. (2005) define a set of axioms for preferences over ambiguous acts, and show that when these axioms are satisfied act f is preferred to act g if and only if

$$\mathbf{E}_p\phi(\mathbf{E}_\pi u \circ f) > \mathbf{E}_p\phi(\mathbf{E}_\pi u \circ g), \tag{1}$$

where u is a von Neumann-Morgenstern utility function, ϕ is an increasing function, and p is a subjective second-order probability over a set Π of probability measures π that the decision maker (DM) deems to be relevant to her decision problem. When ϕ is concave, the DM can be said to be ambiguity averse, i.e. she dislikes mean-preserving spreads in the set of expected utilities implied by her model set Π. To immediately give this model a climatic interpretation, suppose that the choice variable is the level of abatement a of greenhouse gas emissions. Assume that there are m probability models of how abatement affects consumption c. For each model, write the expected utility obtained as a function of a as $EU_m(a)$. To make things concrete one can consider the following:

$$EU_m(a) = \int U(c)\pi_m(c,a)dc - \Lambda(a). \tag{2}$$

In this representation, abatement affects the consumption risks that we face endogenously through the probability distribution $\pi_m(c,a)$, and comes at a utility cost $\Lambda(a)$. Presumably, more abatement increases the probability of high consumption, and decreases the probability of low consumption, but for the moment we will not place any constraints on the distributions π_m.

Now suppose that the DM is ambiguity averse, and that she has smooth ambiguity preferences *a la* Klibanoff et al. (2005). Let ϕ define her ambiguity preferences, i.e. $\phi' > 0, \phi'' < 0$, and p_m be the subjective second order probability of model/prior m. Her objective function V is:

$$V(a) := \sum_m p_m \phi\left(\int U(c)\pi_m(c,a)dc - \Lambda(a)\right). \tag{3}$$

The first order conditions can be written as

$$\Lambda'(a^*) = \sum_m \hat{p}_m(a^*) \int U(c)\frac{\partial \pi_m}{\partial a}(c,a^*)dc \tag{4}$$

where a^* is the optimal abatement level, and we have defined the ambiguity-adjusted second order probabilities $\hat{p}_m$ as:

$$\hat{p}_m(a^*) = \frac{\phi'(EU_m(a^*))p_m}{\sum_n \phi'(EU_n(a^*))p_n}. \tag{5}$$

Equation (4) just says that the marginal costs of abatement should equal the weighted sum of its marginal expected utility benefits over all models, where the weighting factors are just the $\hat{p}_m$. If we look at (5), it is clear that the weighting favours those models that predict low expected utilities, since ϕ' is a decreasing function. Moreover, an increase in ambiguity aversion puts more weight on models with low expected utilities, and less on those with high expected utilities. This can be formalized using the concept of monotone likelihood ratios. Gollier & Gierlinger (2008) prove the following result:

Proposition 1. *Suppose that there are M models, and that the EU_m are ordered such that $EU_1 \leq \ldots \leq EU_M$. Let $\phi_2 = f(\phi_1)$, where f is increasing and concave, and let p^1_m, p^2_m be the ambiguity-adjusted second order probabilities associated with ϕ_1 and ϕ_2 respectively, as given by equation (5). Then p^1_m dominates p^2_m in the sense of the monotone likelihood ratio order, i.e. p^2_m/p^1_m is decreasing in m.*

The proof is very simple - the ratio $p^2_m/p^1_m \propto f'(\phi_1(EU_m))$, where the proportionality constant is independent of m. Since ϕ is increasing, f' is decreasing, and the EU_m are increasing in m, the factor on the right decreases when m increases. This provides a simple characterization of the effect of increased ambiguity aversion on the first order condition. It is important to stress however that the weights $\hat{p}_m$ are endogenous to the optimization problem, as they depend on a^*. Thus translating how these weights change into statements about how optimal abatement changes when ambiguity aversion increases is a non-trivial task in general.

Example 1. In order to get an intuition for how optimal abatement and the ambiguity-adjusted weights on models depend on ambiguity aversion we consider a stylized abatement problem, an extension of an example examined by Gollier (2009). Assume we have a set of risk distributions $\pi_\theta(x)$, indexed by the continuous parameter θ, where x is the magnitude of additive consumption damages from climate change. We can moderate these risks by investing in abatement a. Abatement reduces the magnitude of damages by a multiplicative factor that is linear in a. Abatement comes at a consumption cost that is linear in a, and we are ambiguity averse. The objective function is:

$$V(a) = \int p(\theta)\phi\left(\int U(c_0 + (1 - b_1 a)x - b_2 a)\pi_\theta(x)dx\right) d\theta. \tag{6}$$

Proposition 2. *Assume that*

$$U(c) = -\frac{1}{A}\exp(-Ac) \tag{7}$$

$$\phi(U) = -(-U)^{1+\xi}/(1+\xi) \tag{8}$$

$$\pi_\theta(x) \sim \mathcal{N}(\theta, \sigma) \tag{9}$$

$$p(\theta) \sim \mathcal{N}(\mu, \sigma_0) \tag{10}$$

where $A, \xi, \sigma, \sigma_0, b_1, b_2$ and c_0 are non-negative constants, and the sign of μ is arbitrary. Then when $\mu < (>) - b_2/b_1$ the abatement level a^* that maximizes (6) is increasing (decreasing) in ξ. Moreover, a change in ξ only affects the mean of the ambiguity-adjusted second order probability distribution $\hat{p}(\theta)$, which is decreasing (increasing) in ξ when $\mu < (>) - b_2/b_1$.

Before proving the proposition we unpack the interpretation of the parameters: Our choices for U and ϕ correspond to constant absolute risk aversion (parametrized by A) and constant relative ambiguity aversion (parameterized by ξ) respectively. We have assumed that each of the risks $\pi_\theta(x)$ is a normal distribution, with fixed standard deviation σ, and an uncertain mean θ. The second order distribution of the means θ is also normal, with mean μ and standard deviation σ_0. Note that since x is intended to represent negative climate impacts, we implicitly assume that $\mu < 0$, although this condition is not required in any of our results. Even with $\mu < 0$ however, for each risk distribution there is a non-zero probability that impacts will be positive, since the normal distribution has support on the whole real line – this represents the possibility of climate windfalls. Finally, b_1 determines the effectiveness of abatement (marginal benefits of abatement are just $b_1 x$), b_2 is the marginal cost of abatement, and c_0 is a baseline consumption level. Thus the ratio b_2/b_1 is small when abatement is effective (i.e. cheap and/or strongly moderates risks). We now turn to a proof of the proposition:

Proof. $V(a)$ can be computed explicitly (see Appendix A), and maximized to show that the optimal value of a is

$$a^* = \frac{1}{b_1} + \frac{b_1\mu + b_2}{Ab_1(\sigma^2 + (1+\xi)\sigma_0^2)}. \tag{11}$$

Thus when $\mu < (>) - b_2/b_1$, the optimal value a^* is increasing (decreasing) in ξ. Some standard computations (see Appendix A) then show that

$$\begin{aligned}\hat{p}(\theta) &\sim \mathcal{N}(\mu - \sigma_0^2\xi A(1 - b_1 a^*), \sigma_0^2) \\ &= \mathcal{N}\left(\mu + \frac{\sigma_0^2\xi(b_1\mu + b_2)}{\sigma^2 + (1+\xi)\sigma_0^2}, \sigma_0^2\right)\end{aligned} \tag{12}$$

Thus in this case, the only effect of ambiguity aversion is to shift the mean of the second order probabilities $\hat{p}(\theta)$ downwards (upwards) when $\mu < (>) - b_2/b_1$. Thus when $\mu < -b_2/b_1$, as ambiguity aversion increases, increased weight is placed on those risk distributions with mean $\theta < \mu$, while the weights on risks with $\theta > \mu$ decline. The converse obtains when $\mu > -b_2/b_1$. $\square$

The condition $\mu < -b_2/b_1$ in the proposition is intuitively compelling. It suggests that if mean damages are high enough (i.e. large in absolute value) relative to a measure of the effectiveness of abatement, then ambiguity aversion increases optimal abatement. An increase in ambiguity aversion means that the DM is more averse to spreads in expected utility, so in order to understand the comparative statics we need to understand how a change in a affects the spread in expected utilities. If, on average, an increase in a increases consumption (i.e the argument of U in (6)), the spread in expected utilities will decrease, since by concavity U, and therefore its expected value, becomes less sensitive to variations in consumption when consumption increases. For a fixed value of x, the argument of U increases with a if and only if $x < -b_2/b_1$. Thus our condition $\mu < -b_2/b_1$ may be interpreted as ensuring that on average an increase in a increases consumption, which in turn reduces the spread in expected utilities. Since a reduction of the spread in expected utilities is desirable when ξ increases, it is optimal to increase a. When $\mu > -b_2/b_1$ the converse case obtains, with a decrease in a increasing consumption on average, and thus being desirable when ξ increases.

In the following example we present a comparative statics result in another simplified model of climate change policy under ambiguity. In this example ambiguity preferences are kept completely general, but we use a simpler representation of climate damages and the set of second-order beliefs.

Example 2. Suppose that we have two models of the effect of greenhouse gases on climate:

- Model 1: Climate is not sensitive to anthropogenic emissions.
- Model 2: Climate is sensitive to anthropogenic emissions. Suppose that, conditional on this model being correct, there are two climate states, a high damage state H, and a low damage state L. Abatement increases the probability of state L materializing.

The DM attaches a default utility level u_0 to today's climate. Let $u_0 - d_H$ be the utility level in the high climate damages state, and $u_0 - d_L$ be the utility level in the low damages state, with $d_H > d_L$. The DM must decide the level of abatement, a. The utility cost of abatement is given by a function $\Lambda(a)$, and let $\pi(a)$ be the endogenous probability of the low-damages state occurring as a function of abatement. Clearly we need $\pi'(a) > 0$, $\Lambda'(a) > 0$. Finally, let p be the subjective second-order probability of model 2 being correct, and ϕ be the ambiguity aversion function, with $\phi' > 0, \phi'' < 0$. The planner's objective function is then:

$$\begin{aligned} V &= (1-p)\phi(u_0 - \Lambda(a)) + p\phi\left(\pi(a)(u_0 - d_L - \Lambda(a)) + (1-\pi(a))(u_0 - d_H - \Lambda(a))\right) \\ &= (1-p)\phi(u_0 - \Lambda(a)) + p\phi\left(u_0 - \Lambda(a) + \pi(a)(d_H - d_L) - d_H\right). \end{aligned} \tag{13}$$

Proposition 3. *In the model defined by equation (13), assume that $\pi''(a) < 0$ and $\Lambda''(a) > 0$. Then an increase in ambiguity aversion increases the optimal level of abatement.*

Proof. Differentiating (13) with respect to a, we can write the first order condition as:

$$(d_H - d_L)\pi'(a) = K(a)\Lambda'(a) \tag{14}$$

where we define

$$K(a) := \frac{1-p}{p} \frac{\phi'(u_0 - \Lambda(a))}{\phi'(u_0 - \Lambda(a) + \pi(a)(d_H - d_L) - d_H)} + 1 \tag{15}$$

Notice that the effect of ambiguity aversion is completely determined by its effect on K. The proof now proceeds in two steps:

(i) Let $Q = \{a | (d_H - d_L)\pi'(a) > \Lambda'(a)\}$. Then $K'(a) > 0$ on Q. This follows by direct differentiation of $K(a)$, and the properties of ϕ. Notice that when $K'(a) > 0$, and $\Lambda''(a) > 0$, the right hand side of (14) is a product of two increasing functions, and is thus itself increasing in a.

(ii) An increase in ambiguity aversion decreases $K(a)$ for all $a \in Q$. To see this, let ϕ_1 be an ambiguity aversion function, and $\phi_2(x) = f(\phi_1(x))$ be a concave transformation of ϕ_1, i.e. ϕ_2 exhibits more ambiguity aversion that ϕ_1. Then for any x, y:

$$\frac{\phi_2'(x)}{\phi_2'(x-y)} = \frac{f'(\phi_1(x))}{f'(\phi_1(x-y))} \frac{\phi_1'(x)}{\phi_1'(x-y)} < \frac{\phi_1'(x)}{\phi_1'(x-y)} \tag{16}$$

The inequality follows from the fact that $\phi_1(x) > \phi_1(x-y)$, and thus $f'(\phi_1(x)) < f'(\phi_1(x-y))$, since $f'' < 0$. So, if we apply this result to the quotient in the definition of K, any increase in ambiguity aversion must necessarily decrease K for all a, and thus for all $a \in Q$.

We now combine these two results. Notice first that $K(a) > 1$, thus the solution to the optimization problem, if it exists, lies in the set Q, for any ϕ. In Q, $\pi'(a)$ is decreasing by assumption, and $K(a)\Lambda'(a)$ is increasing, as we demonstrated in (i). Thus a reduction in $K(a)$ for all $a \in Q$ must necessarily increase the value of a that solves (14). $\square$

3 Evaluating dynamic abatement pathways under ambiguity

The preceding section abstracted the climate change abatement problem to a high level. Perhaps most importantly, the models examined thus far have all been atemporal. While static models are useful for gaining an intuition for the new effects that ambiguity aversion introduces into familiar problems, they are of limited use for deriving results that are meaningful for climate policy. The abatement problem is in its essentials a dynamic decision problem, in that it requires us to trade off near-term costs against uncertain long-term benefits. This section examines how ambiguity over the future benefits of abatement affects the welfare analysis of alternative climate policies. We first introduce a dynamic extension of the smooth ambiguity model of Klibanoff et al. (2005), derived in Klibanoff et al. (2009). Doing so immediately raises issues of how dynamic consistency and learning are incorporated in such a model, and we discuss these. Second, we make an empirical application of the Klibanoff et al. (2009) model to climate policy, using the DICE integrated assessment model (Nordhaus, 2008) to generate consumption streams given choices of abatement strategy, in the face of ambiguous information about the climatic response to greenhouse gas emissions (i.e. represented by the climate sensitivity parameter).

Klibanoff et al. (2009) obtain a representation of preferences over time and state dependent acts, i.e. contingent plans that map the nodes of a decision tree into consumption streams. If we let $s^t = (x_1, \ldots, x_t) \in \Gamma$ denote a decision node, where $x_\tau \in \mathcal{X}_\tau$ is an observation at time τ and Γ is the set of all nodes, then a generic plan f maps s^t into consumption. Klibanoff et al. (2009) show that preferences over plans that satisfy consequentiality, dynamic consistency, and further axioms that are similar to those employed in the static representation result, can be represented by

$$V_{s_t}(f) = u(f(s^t)) + \beta\phi^{-1}\left[\int_\Theta \phi\left(\int_{\mathcal{X}_{t+1}} V_{(s^t,x_{t+1})}(f)d\pi_\theta(x_{t+1}; s^t)\right) dp(\theta|s^t)\right], \tag{17}$$

where $\theta \in \Theta$ indexes the set of alternative probability models, and β is a discount factor. They refer to this representation as the *recursive form*, for obvious reasons. Although the focus of their paper is on achieving this result, Klibanoff et al. (2009) also derive an alternative, distinct, preference representation, which they call the *reduced form*:

$$\hat{V}_{s_t}(f) = u(f(s^t)) + \beta\phi^{-1}\left[\int_\Theta \phi\left(\sum_{s\in\Gamma\cap\{s^t\}} \pi_\theta(s|s^t) \sum_{\tau\geq t+1} \beta^{\tau-(t+1)}u(f(s)(\tau))\right) dp(\theta|s^t)\right]. \tag{18}$$

The difference between these two representations arises from the different requirements they impose on consistency between so-called first- and second-order acts (see Klibanoff et al., 2009, for details). The recursive form requires consistency only for 'one-step-ahead continuation plans', while the reduced form requires consistency for all continuation plans. It turns out that the recursive form is dynamically consistent, while the reduced form is not. To see why notice that the reduced form evaluates all future consumption streams by averaging over the *current* second-order uncertainty distribution $p(\theta|s^t)$. However it should be clear that this is not the distribution that we would use to compute expectations at a future decision node, since in reaching that future node new observations would be made that allow us to update the second order probabilities – this constitutes a violation of dynamic consistency. In fact, the reduced form representation requires DMs to act as if all the second order uncertainty will be resolved in the next time period. The recursive formulation respects dynamic consistency, since it only uses the current information to average over continuation values, which are themselves recursively defined in terms of averages over future (updated) information. It is thus capable of representing preferences that coherently account for the persistence of ambiguity. The conflict between these two distinct representations is a new feature of inter-temporal choice which arises from the desire to simultaneously represent ambiguity sensitive preferences and respect consequentialism (Klibanoff et al., 2009, p. 946). Note that when ϕ is affine, i.e. the DM is ambiguity neutral, the two representations are identical; this explains why these issues are not encountered in the standard risk case.

To make the discussion concrete, we will now use the DICE model to analyse the empirical effect of ambiguity on climate policy, focusing on ambiguity over the climate sensitivity. We will not be using the DICE model to solve for optimal abatement policies, but will rather use it to generate consumption streams that depend on exogenous policy settings, and a value for the climate sensitivity. While the recursive form (17) can be treated as a Bellman-like equation that can be used to solve for optimal abatement policies once a noise distribution for temperature observations has been specified, this approach entails considerable computational complications (see e.g. Kelly & Kolstad, 1999). This is due to the high dimensionality of the state space when there are multiple distributions for the climate sensitivity that need to be tracked and updated as observations of temperature are made. We hope to take up this task in future work. Evaluating exogenous policies is a desirable step – emissions pathways may, for example, be subject to political constraints that prevent them from being 'optimal', yet we may still wish to evaluate their effects on welfare, and determine how those effects depend on our attitude to ambiguity.

If we denote the climate sensitivity by S, the exogenous savings rate by $\sigma(t)$, and the exogenous abatement effort by $a(t)$, then we view DICE as the following function:

$$\mathrm{DICE}\,(S;\sigma(t),a(t)) = c(t), \tag{19}$$

where $c(t)$ is a consumption stream. We can now compute this function for a variety of values of S, holding $\sigma(t)$ and $a(t)$ constant, to see how the consumption stream depends on the climate sensitivity. Before specifying our choice of the controls, a few words are in order about the DICE model, which is explained in full detail in Nordhaus (2008). DICE is a well known integrated assessment model of the connections between economic activity and climate change. A standard Ramsey-Cass-Koopmans growth model with aggregate capital and labour inputs is linked to climate change through emissions of greenhouse gases, which cause global warming and, with a lag, reduce output by means of a reduced form 'damage function' (more on this later). This damage function incorporates assumptions about adaptation to climate change, which can reduce output losses, leaving the representative agent with the choice of how much to invest in emissions abatement $a(t)$, as well, of course, as how much to invest in the composite capital good. The model includes many economic and climate parameters, all of which are at least to some extent uncertain. However, for the sake of tractability, we focus on uncertainty (specifically ambiguity) about the climate sensitivity S. This is justified, since S is known to be one of the most, if not indeed *the* most, uncertain parameters in integrated assessment of climate change, and one that has a potentially strong influence on the value of abatement strategies (Weitzman, 2009). Moreover, as we have argued above, it is a parameter about which our knowledge is appropriately represented as ambiguous[5]. Future work could also, where appropriate, treat knowledge about other parameters as ambiguous.

In our empirical results below, we pick specific values for the controls $\sigma(t)$ and $a(t)$. For simplicity, we assume that the savings rate $\sigma(t)$ is a constant 22%. In order to fix the abatement effort $a(t)$, we consider three emissions abatement scenarios. Abatement effort is represented in DICE by the emissions control rate, a number between 0 and 1, which controls the emissions intensity of gross economic output (i.e. before climate damages are incurred). When the control rate is $a(t)$, a fraction $1 - a(t)$ of gross output contributes to emissions. Our three scenarios for the control rate are a 'Business as usual' scenario (hereafter BAU), a scenario that limits the atmospheric concentration of CO_2 to twice its pre-industrial level (560 parts per million, hereafter referred to as the 2 CO_2 scenario), and a more aggressive abatement scenario that limits the concentration of CO_2 to only one-and-a-half times its pre-industrial level (420ppm, hereafter referred to as the 1.5 CO_2 scenario)[6]. Both of the abatement scenarios have been prominent in recent international negotiations about climate policy. The emissions control rates corresponding to our three scenarios are depicted in Figure 2.

Suppose now that we have a set of M distributions $\{\pi_m(S)\}$ for the climate sensitivity S, indexed by m, with initial second-order probability weights given by p_m. How can we compare the welfare effects of the two abatement pathways? Ideally we would like to use the recursive preference representation (17), however we immediately run into difficulties. In order to apply this representation we need to know how the first and second order probabilities $(\pi_m(S), p_m)$ change over time as more information is revealed. Yet this information is not part of our primitive inputs to the model – all we know is the values of our plan $f(s^t)$, but not how the sequence of realized states $\{x_t\}_{t=1...\infty}$ affects beliefs $\pi_m(S)$ and p_m. Moreover, strictly speaking, the manner in which we make use of the DICE model implies that *all* uncertainty (i.e. risk and ambiguity) is resolved immediately. This is so since in our setup the DICE model is a deterministic function. This implies that as soon as a single time period has elapsed the DM can use the known model equations to work back from the changes in the state variables to deduce the exact value of the

[5]In fact, there are good reasons to believe that the upper tails of the various distributions of S estimated in the climate science literature are not well constrained by observations, and depend heavily on scientists' subjective choice of prior (Allen et al., 2006). We are arguing that scientists' prior beliefs about S are not sufficiently structured for them to be representable by a unique probability distribution; hence unique posteriors also do not exist.

[6]Note that the control rates associated with the two abatement scenarios are designed so that they achieve the specified stabilization targets in an idealized run of DICE in which damages are zero for temperatures below the stabilization target, and rise sharply to very high values above the target. This is the method used by Nordhaus (2008) to generate controls that achieve a given stabilization target, however it should be born in mind that these controls will not in general achieve this target when they are used as inputs to DICE for alternative damage functions, or values of S. Since all we require for our purposes is a plausible choice of controls, we need not concern ourselves too much with whether they achieve a given stabilization target.

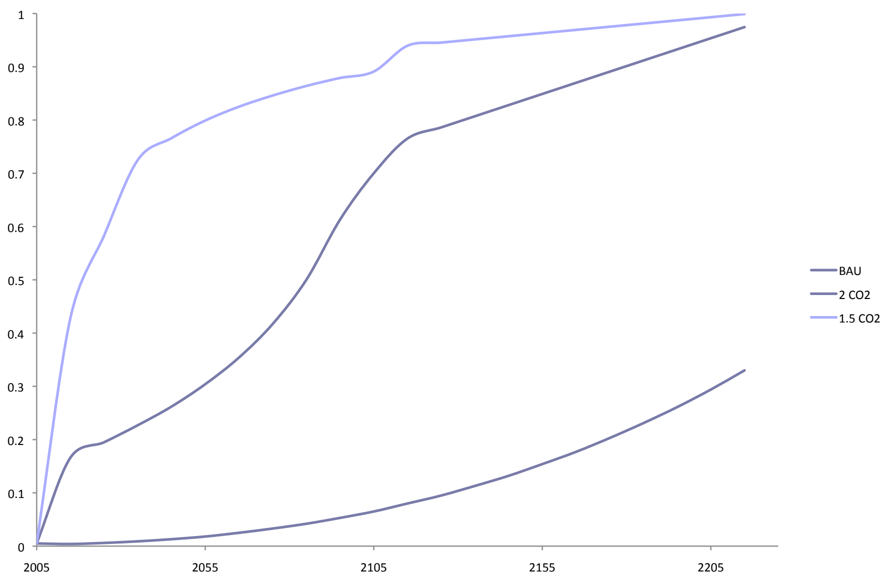

Figure 2: Emissions control rates for our sample abatement scenarios.

climate sensitivity[7] . Clearly this does not map onto the scientific situation in reality, in which uncertainty about the climate sensitivity is likely to persist for some time.

There are two resolutions to these difficulties. The first is to use the reduced form representation to evaluate welfare. As mentioned above, this implies that we are evaluating policies today as if all ambiguity were to be resolved in the very next time step. This has the disadvantage of not respecting dynamic consistency (except of course in the unlikely event that we believe ambiguity really will be resolved after one time step), but has the advantage of being consistent with our usage of the DICE model as a deterministic function. The reduced form approximation can be made more realistic by increasing the length of the time step, so that an *a priori* plausible amount of time passes before ambiguity is resolved. Thus in our computations below we increase the length of the time step in DICE to three decades. Since DICE's initial period is 2005, ambiguity is assumed to be resolved in 2035 for the reduced form welfare measure.

The second resolution is to use the recursive form, and assume an *ad hoc* model for how ambiguity and risks are updated over time. This has the advantage of allowing for a persistent (albeit arbitrary) dynamic representation of ambiguity, but requires us to pretend that the consumption streams at our disposal are divorced from the model that generated them, i.e. we pretend that we do not know the model equations, so that uncertainty is not resolved in the first time step. An extreme version of this strategy would be to assume that no information is obtained at any point in time, i.e. ambiguity is perfectly persistent, and the DM never updates her beliefs.

Our strategy will be to use both the reduced form and the recursive form with perfectly persistent ambiguity to compute welfare measures. It seems reasonable to believe that these two approaches provide meaningful bounds on the value of an abatement project. Any model which

[7] This is so provided the model's state equations are 'invertible' (i.e. we can infer parameter values from sequential realizations of the state variables). The immediate resolution of uncertainty in our model is due to the fact that there are no noise terms in the state equations.

admits updating of probabilities must fall somewhere between these two extremes[8].

For the sake of argument we assume equal second order weights on each of the distributions for climate sensitivity depicted in Figure 1, and choose u and ϕ both to be isoelastic functions (constant relative risk and ambiguity aversion respectively):

$$u(c) = \frac{c^{1-\eta}}{1-\eta}, \tag{20}$$

$$\phi(u) = \begin{cases} \frac{u^{1-\xi}}{1-\xi} & \eta < 1 \\ \frac{-(-u)^{1+\xi}}{1+\xi} & \eta > 1 \end{cases}, \tag{21}$$

where η (ξ) is the coefficient of relative risk (ambiguity) aversion. We then use DICE to generate consumption streams for climate sensitivities in the range $[1, 10]$[9], and compute welfare measures for each of the two representations of ambiguity and abatement strategies. The welfare measure is the net present consumption-equivalent value of our abatement strategies relative to BAU, as a fraction of 2005 GDP. We define the effective net present value per capita (Δ) of the abatement pathway relative to the BAU pathway through

$$V_{ABATE} - V_{BAU} := n_0[u(c_0 + \Delta) - u(c_0)] \tag{22}$$

where n_0 is the 2005 global population, c_0 is 2005 global consumption per capita, and $ABATE$ denotes either the 1.5 CO_2 or 2 CO_2 abatement scenario. In all our computations, we set the annual discount rate on utility to 0.1%, consistent with the original analysis of Nordhaus (2008) and recent viewpoints by, for example, Stern (2008), Dasgupta (2008), and Heal (2009). In our base case, we set the coefficient of relative risk aversion (η) to 2, and we use the standard DICE damage function,

$$\Omega(t) = \frac{1}{1 + \alpha_1 T(t) + \alpha_2 T(t)^2}, \tag{23}$$

where T is the increase in global mean temperature above the pre-industrial level, and α_1 and α_2 are coefficients. The damage from warming as a fraction of GDP is $1 - \Omega(t)$, which with the default calibration of α_1=0 and α_2=0.0028, gives damages of 1.7% of GDP for 2.5°C warming and 6.5% for 5°C warming.

Figure 3 plots $n_0\Delta/GDP_{2005}$ as a function of the coefficient of relative ambiguity aversion (ξ) for the 1.5 CO_2 and 2 CO_2 abatement strategies. First consider the solid lines, which plot the relationship for the base case of η=2 and default damages. The figure shows that the NPV of abatement increases as ξ increases. Focussing on the recursive form with perfectly persistent ambiguity (solid blue), the NPV of the 1.5 CO_2 strategy increases from 0.07% of GDP when ξ=0 to approximately 0.25% of GDP when ξ=80. Under the 2 CO_2 strategy recursive NPV increases from 0.46% of GDP when ξ=0 to 0.62% of GDP when ξ=80.

That increasing ambiguity aversion has this effect on the NPV of abatement essentially reflects

[8]The fact that the recursive form is equivalent to the reduced form when the learning process employed in the recursive model resolves all uncertainty in one time step is a trivial consequence of the fact that the two representations agree on their ranking of one-step-ahead continuation plans.

[9]The simple climate model built into DICE breaks down for $S < 1$, so we are forced to truncate the distributions in Figure 1, applying a lower bound value of 1. We do not lose much information in this way, since inspection of Figure 1 shows that there is a very low probability that $S < 1$, and that the upper tail is of much greater importance.

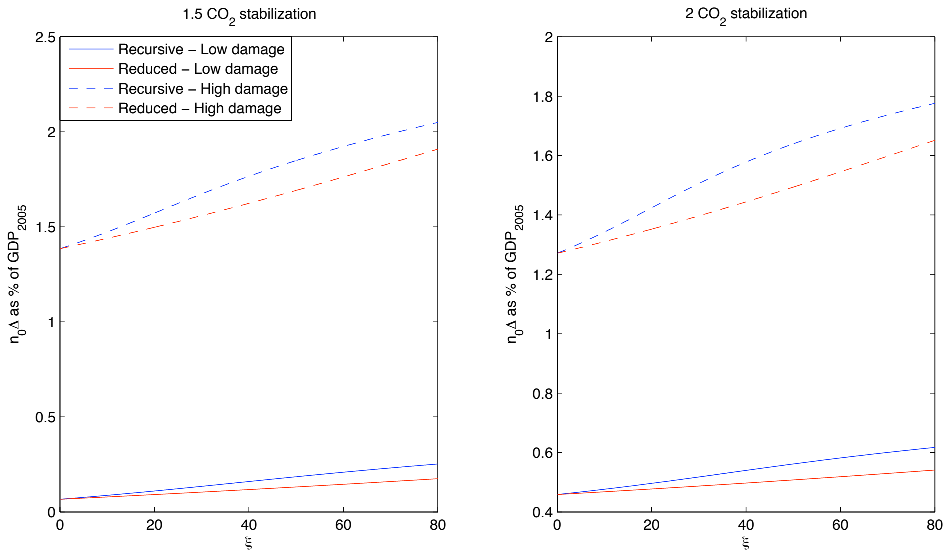

Figure 3: Present consumption-equivalent welfare benefit of a 1.5 CO_2 (420ppm) and (2 CO_2) (560ppm) abatement pathway relative to BAU, as a function of the coefficient of relative ambiguity aversion (ξ). The solid lines plot the relationship for default ('low') damages, and the dashed lines do so for high damages. Estimates based on the reduced form model are plotted in red, while estimates based on the recursive model with perfectly persistent ambiguity are plotted in blue. $\eta = 2, \delta = 0.1\%$ for these simulations.

the forces at play in our second stylised example above. Climate is sensitive to emissions, so the higher are emissions, the more likely it is that high damages result. An increase in ambiguity aversion places more weight on models with lower expected utility, or in other words models in which damages are higher. In these models, costly emissions abatement is of more value as it avoids greater amounts of climate damage. In Figure 3 the base-case relationships appear close to linear for the range of ξ that we consider. However it is easy to see that, were it computationally feasible[10] to further increase ξ, the net present value of the abatement strategy would eventually asymptote to a constant value as ξ tends to infinity[11].

The dashed lines in Figure 3 consider the relationship between the value of abatement and ambiguity aversion for a higher calibration of DICE's damage function. We consider higher damages both to test the sensitivity of our results to the damage specification, and since DICE's default calibration has been argued to be implausibly low (Ackerman et al., 2009; Weitzman, 2009). We double α_2 to 0.0056, so that 2.5°C warming causes damage amounting to 3.4% of GDP and 5°C warming causes damage totalling 12.3% of GDP. Both the responsiveness of NPV to ambiguity aversion and the spread between the recursive and reduced form increase for this case. That NPV is more sensitive to ambiguity aversion for high damages than for low damages is a consequence of the fact that the spread in expected utilities over the different models increases when damages are

[10] The values of ξ it is feasible to consider numerically are limited by the accuracy of floating point arithmetic.

[11] The limiting values as $\xi \to \infty$ are not the same for the recursive and reduced form welfare measures. The limiting reduced form NPV depends only on the model with the lowest discounted expected utility, while the limiting recursive form NPV depends on the discounted sum of the lowest expected utility of *any* model at each point in time.

high; this is so since higher damages imply that consumption levels are more sensitive to the value of S, which in turn implies that expected utilities are more sensitive to the distribution assumed for S. As a consequence of the same effect, the concavity of the relationship between NPV and ambiguity aversion is also easier to see in the case of high damages, since the gap between the lowest expected utility and those of the other models increases as damages increase.

Figures 4 and 5 consider the relative importance of ambiguity aversion compared with risk aversion, and how the two preferences interact with one another. In order to understand the

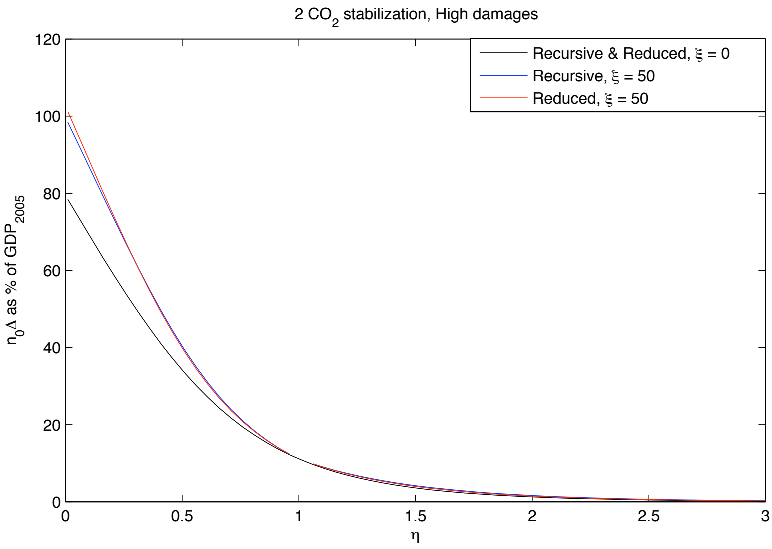

Figure 4: Present consumption-equivalent welfare benefit of the 2 CO_2 abatement pathway relative to BAU, as a function of the coefficient of relative risk aversion (η). The black line plots the relationship when the coefficient of relative ambiguity aversion $\xi = 0$, while the blue and red lines plot the relationship when ξ=50 for the recursive and reduced form welfare measures respectively.

qualitative features of these figures we contrast them with the expression for the certainty equivalent social discount rate (ρ) under ambiguity aversion, derived in Traeger (2009) and Gollier & Gierlinger (2008). These derivations assume isoelastic forms for u and ϕ, and that consumption grows at an uncertain rate $g \sim \mathcal{N}(\theta, \sigma)$, where the mean growth rate θ is itself uncertain and distributed according to a second-order probability distribution, $\theta \sim \mathcal{N}(\mu, \sigma_0)$. It can then be shown that

$$\rho = \delta + \eta\mu - \frac{\eta^2}{2}(\sigma^2 + \sigma_0^2) - \xi|1 - \eta^2|\frac{\sigma_0^2}{2}, \tag{24}$$

where δ is the utility discount rate. The first two terms in this expression are familiar from the standard Ramsey formula for the social discount rate under certainty, and capture inter-temporal substitution effects. The third term is the standard correction due to the uncertainty in the growth rate (note that the variance of the growth rate is the variance of the composite distribution that arises from the combination of uncertainty about g and θ). The final term is a new addition due to ambiguity aversion.

Since differences in the expected utilities of abatement and BAU pathways only manifest significantly in the distant future (when damages are large), it is intuitive that the social discount

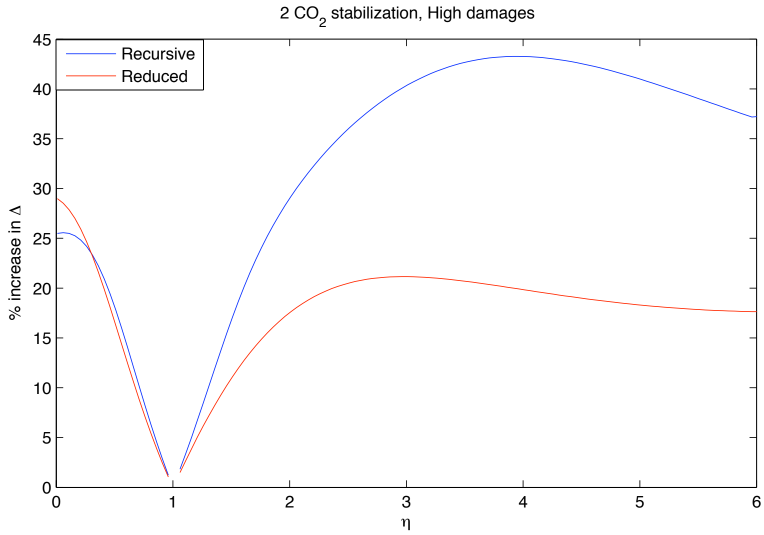

Figure 5: Percentage change in Δ for $\xi = 50$, relative to $\xi = 0$.

rate may play a useful role in explaining our results. If ρ is large, we expect Δ, the NPV per capita of abatement relative to BAU, to be small. Similarly, those values of η for which ρ is highly sensitive to ξ are also likely to be the values of η for which we see a significant effect of ambiguity aversion on Δ. Although the assumptions about the first- and second-order uncertainty on consumption growth rates used to derive (24) do not map neatly onto our empirical application, the expression nevertheless provides useful qualitative insights.

First notice that ρ is decreasing in ξ – this is consistent with our finding in Figure 3 that Δ is increasing in ξ. Now consider the case $\eta = 0$, for which $\rho = \delta - \frac{1}{2}\xi\sigma_0^2$. It is clear that in this case an increase in ξ can have a dramatic impact on the valuation – in fact if ξ is large enough ρ can be negative, thus placing increasing weight on the future, which should lead to large values of Δ. This is reflected in Figure 4, where we see a large difference between $\Delta_{\xi=0}$ and $\Delta_{\xi=50}$. Thus ambiguity aversion has a substantial absolute effect on welfare for η close to zero. Now consider the case $\eta = 1$. It is clear that in this case ρ is independent of ξ, implying that ambiguity aversion has no effect on welfare calculations. This is also reflected in Figures 4 and 5, where we see that $\Delta_{\xi=0}$ and $\Delta_{\xi=50}$ coincide at $\eta = 1$. Finally, notice that when η becomes large, Δ tends to zero in Figure 4. This suggests that the inter-temporal substitution terms in the expression for ρ dominate the risk and ambiguity terms in our simulations. For this to be so, the mean growth rate in consumption should be significantly larger than the variance of consumption growth within a given model, and the variance of mean consumption growth rates between models. This is born out by our simulation data, for which we find inter- and intra-model variances in consumption growth to be several orders of magnitude smaller than the mean growth rate of consumption[12]. Note that, although Figure 4 might be taken to imply that the effect of ambiguity aversion on welfare is decreasing in η, Figure 5 reveals the η-dependence of the welfare measures to be more

[12]Neither growth rates, nor their variances, are constant in our DICE simulations, with effective annual growth rates tending to fall from approximately 2% to 1% over our 200 year simulation horizon. However the ratio of inter- and intra-model growth rate variances to mean growth rate is smaller than 10^{-6} for every time step.

complex. Nevertheless, it is clear from our results that welfare is more sensitive to the choice of η than it is to the choice of ξ for our set of sensitivity distributions $\{\pi_m(S)\}$.

A further interesting feature of Figure 4 is that the difference between the recursive and reduced form welfare measures is not very large in absolute terms. Thus, although the assumptions these two welfare measures make about the resolution of uncertainty are poles apart, they place tight bounds on the absolute effect of ambiguity aversion on welfare for our simulations. Perhaps somewhat surprisingly, Figure 5 also makes it clear that the ranking of recursive and reduced form welfare measures is not independent of the value of η. In general the ranking of these measures as a function of η is highly complex, and depends on the empirical details of the consumption streams, the distribution set $\{\pi_m(S)\}$, and the value of ξ. For a heuristic explanation of the origins of the complexity of this dependence, see our discussion of a simplified model in Appendix B. Of course, the ranking of these measures is of no consequence if our primary concern is to place meaningful upper and lower bounds on the true welfare evaluation – all we need to know is that the true value lies between these two measures.

4 Discussion and conclusions

This paper aimed to provide insight into how ambiguous beliefs, and aversion to ambiguity, affect the welfare analysis of climate change abatement policy. We have argued that our knowledge of the climate system is not of sufficiently high quality to be described with unique probability distributions, and that formal frameworks that account for aversion to ambiguity are both normatively legitimate, and provide a more accurate representation of our state of knowledge and our preferences. The paper investigates how such preferences, as represented by the smooth ambiguity model, affect optimal abatement policy in some simple static cases, and explores the conditions under which ambiguity aversion has a significant effect on the welfare evaluation of exogenously specified dynamic abatement policies.

Several lessons were learned from our static models. First, we showed that ambiguity aversion has a simple effect on the ambiguity-adjusted second-order probabilities on models at the level of the first order conditions, with increases in ambiguity placing more weight on models with low expected utilities. The ambiguity-adjusted model probabilities are however endogenously determined, implying that it is not a simple matter to reason from changes in model probabilities to changes in optimal abatement. We presented an example in which it was possible to compute optimal abatement, and the endogenous second-order probabilities, explicitly, and derived a condition on the effectiveness of abatement and mean extent of climate damages which ensures that optimal abatement is increasing in the coefficient of relative ambiguity aversion. When this condition is satisfied the second order probabilities also exhibit a simple dependence on ambiguity aversion, with their mean monotonically decreasing in the coefficient of relative ambiguity aversion. Finally, we presented a model which used a more general representation of ambiguity preferences, and a simpler representation of climate damages, and presented sufficient conditions for optimal abatement to increase when the decision maker's ambiguity preferences undergo an arbitrary concave transformation.

While the static models provide an insight into how ambiguity averse preferences differ from standard risk aversion, and yield suggestive comparative statics results, their relevance to assessing realistic climate policies is limited, as they do not account for their fundamentally dynamic nature. We thus extended our analysis to the evaluation of dynamic abatement policies by using the inter-temporal version of the smooth ambiguity model. We defined two welfare measures based on extremal assumptions about the resolution of ambiguity – a recursive measure in which ambiguity is perfectly persistent, and a reduced form measure in which ambiguity resolves after a single time step – and used these to place bounds on the ambiguity averse welfare evaluation. Using the well-known DICE integrated assessment model, we computed these welfare measures for the case in which only the climate sensitivity parameter is assumed to be ambiguous. Beliefs about the climate sensitivity were represented by 20 estimated distributions from the scientific literature, and each distribution was assigned an equal second-order probability.

Using the above specification we showed that the consumption equivalent NPV of abatement relative to business as usual is increasing in the coefficient of relative ambiguity aversion. In addition, the sensitivity of NPV to ambiguity preferences increases when the assumed damages from climate change increase. We also investigated the interaction between risk and ambiguity preferences in determining the NPV of abatement. When the coefficient of relative risk aversion (η) is 1 the NPV is insensitive to ambiguity aversion. As η decreases from 1 to 0, NPV becomes increasingly sensitive to ambiguity aversion, with the greatest absolute effect obtaining for $\eta = 0$. As η increases above 1 the absolute effect of ambiguity aversion is small due to the dominance of the inter-temporal substitution effect of η, however the relative effect of ambiguity aversion on NPV is still significant. In general we conclude that the NPV of abatement relative to business as usual is more sensitive to η than to the coefficient of relative ambiguity aversion over the range of parameters we examined. This is largely explained by a single fact – the spread about the mean of the average consumption growth rates over our set of distributions for the climate sensitivity is small for all time. The inter-model spread of average consumption growth rates depends largely on two factors – the magnitude of the climate damages, and how different the alternative distributions for the climate sensitivity are from one another. Higher damages would make growth rates more sensitive to the value of the climate sensitivity, which in turn will heighten the effect of differences between sensitivity distributions on the inter-model spread in average growth rates.

The results presented in this paper are subject to several limitations and qualifications. We have focussed on ambiguity over the climate sensitivity, however there are many other parameters in climate-economy models about which we may have ambiguous beliefs. The parameters of the damage function are especially likely candidates. In addition, our calculations in the dynamic analysis assumed equal second order probabilities on each of the empirical distributions for the climate sensitivity. The smooth ambiguity model's preference representation does not provide guidance as to how these probabilities should be chosen, any more than Savage's results suggest a 'correct' prior. A possible justification for our assumption would to appeal to a second-order principle of insufficient reason. There are however many well-known difficulties with this principle[13], and we do not believe that it provides a satisfactory method for choosing second-order probabilities. Rather, these probabilities should be treated as primitive components of our beliefs, and are thus subject to rigorous elicitation. Our choice of equal probabilities reflects our inability to discern differences in the quality of the various empirical estimates of the climate sensitivity

[13]Perhaps most famously, it is not 'reparameterization invariant', a point made by Keynes (1921). This is not to say that non-informative priors that are reparameterization invariant, i.e. Jeffreys priors, would provide a satisfactory alternative.

distribution – no doubt experts on these estimates would advocate alternative specifications.

At a more fundamental level, the dependence of the smooth ambiguity model's preference representation on second-order probabilities does leave us a little uneasy. Suppose that our beliefs about the veracity of the models in our set are not sufficiently well structured for us to be able to define unique second-order probabilities. Are we then committed to an infinite regress of nested preferences? While the separation between tastes and beliefs that the smooth ambiguity model achieves is a desirable property, it may not be justified in situations of deep uncertainty. The very general representation obtained in Schmeidler (1989) provides an alternative, but suffers from problems of its own[14]. In addition, we have assumed that our model's state-space accurately describes the evolution of the climate-economy system, i.e. that there are no 'surprises'. This seems a strong assumption, give known inadequacies in our understanding of the climate system, not to mention economic aberrations. Decision theories that account for these deficiencies in our understanding have been proposed (e.g. Gilboa & Schmeidler, 1995), although it is as yet unclear whether they can be usefully applied in the climate change context.

Despite these limitations, we have shown that under certain conditions ambiguity aversion can have a significant effect on both optimal abatement, and the welfare evaluation of exogenous policies. This suggests the importance of this line of research for designing climate solutions that are true reflections of our preferences and state of knowledge.

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[14]These include the fact that the representation theorem depends on an explicit 'uncertainty aversion' axiom. One would hope that such a behavioural constraint would be an optional special case of the representation (much as risk aversion is in expected utility theory), rather than a primitive requirement. In addition, Epstein (1999) has argued that this representation is neither necessary nor sufficient to explain ambiguity averse decisions in Ellsberg-type experiments.

A Calculations for Example 1

$$V(a) = \int p(\theta)\phi\left(\int U(c_0 + (1-b_1a)x - b_2a)\pi_\theta(x)dx\right)d\theta.$$

where we assume that

$$\begin{aligned}
p(\theta) &\sim \mathcal{N}(\mu, \sigma_0) \\
\pi_\theta(x) &\sim \mathcal{N}(\theta, \sigma) \\
U(c) &= -\frac{1}{A}\exp(-Ac) \\
\phi(U) &= -(-U)^{1+\xi}/(1+\xi).
\end{aligned}$$

Using the fact that

$$\int e^{\lambda x}\pi(x)dx = e^{\lambda(\theta+\frac{1}{2}\lambda\sigma^2)}$$

for any constant λ, and any $\pi(x) \sim \mathcal{N}(\theta, \sigma)$, one can compute all the integrals in the expression for $V(a)$ to find that

$$V(a) \propto -\exp\left(-A(1+\xi)\left[c_0 - b_2a + (1-b_1a)\mu - \frac{1}{2}A(1-b_1a)^2(\sigma^2 + (1+\xi)\sigma_0^2)\right]\right).$$

$V(a)$ is maximized when the term is square brackets above is maximized. The solution is

$$a^* = \frac{1}{b_1} + \frac{b_1\mu + b_2}{Ab_1(\sigma^2 + (1+\xi)\sigma_0^2)}.$$

Now the ambiguity adjusted distribution of the means θ is:

$$\hat{p}(\theta) \quad = \quad \frac{p(\theta)\phi'\left(\int U(c_0 + (1-b_1a^*)x - b_2a^*)\pi_\theta(x)dx\right)}{\int p(\theta)\phi'\left(\int U(c_0 + (1-b_1a^*)x - b_2a^*)\pi_\theta(x)dx\right)d\theta}$$

Consider the numerator. $\phi'(U) = (-U)^\xi$, so this is just

$$A^{-\xi}p(\theta)\exp\left(-A\xi\left[c_0 - b_2a^* + (1-b_1a^*)\theta - \frac{1}{2}A\sigma^2(1-b_1a^*)^2\right]\right).$$

All the terms that do not depend on θ are going to get divided out, so the important part of the numerator is

$$p(\theta)\exp\left(-A\xi(1-b_1a^*)\theta\right).$$

Because $p(\theta)$ is a normal distribution, we can multiply the exponential into the normal PDF. Once again, we can neglect everything that does not depend on θ, as it will get divided out:

$$
\begin{aligned}
p(\theta)\exp\left(-A\xi(1-b_1a^*)\theta\right) &\propto \exp\left(-\frac{(\theta-\mu)^2}{2\sigma_0^2}\right)\exp\left(-A\xi(1-b_1a^*)\theta\right) \\
&\propto \exp\left(-\frac{1}{2\sigma_0^2}\left[\theta^2 - 2\theta(\mu - A\sigma_0^2\xi(1-b_1a^*))\right]\right) \\
&\propto \exp\left(-\frac{\left[\theta - (\mu - \sigma_0^2\xi A(1-b_1a^*))\right]^2}{2\sigma_0^2}\right)
\end{aligned}
$$

In the last step we have just completed the square. This expression looks like another normal PDF, only with an adjusted mean. Since we know that $\hat{p}(\theta)$ will be properly normalized, we can conclude that

$$\hat{p}(\theta) \sim \mathcal{N}(\mu - \sigma_0^2\xi A(1-b_1a^*), \sigma_0^2).$$

B Ranking recursive and reduced form welfare measures

In order to gain an intuition for why the ranking of recursive and reduced form welfare measures has a complex dependence on η, consider a 3-period model, and assume that beliefs are described by only two different probability models. Let $U(0)$ be the known utility at time 0, $EU_i(t)$ be the expected utility in model $i \in \{1,2\}$ at time $t \in \{1,2\}$, and p be the second-order probability on distribution 1. The two welfare measures are given by:

$$
\begin{aligned}
V_{reduced} &= U(0) + \beta\phi^{-1}\left(p\phi(EU_1(1) + \beta EU_1(2)) + (1-p)\phi(EU_2(1) + \beta EU_2(2))\right) \\
V_{recursive} &= U(0) + \beta\phi^{-1}\left(p\phi(EU_1(1) + \beta X) + (1-p)\phi(EU_2(1) + \beta X)\right)
\end{aligned}
$$

where

$$X := \phi^{-1}\left(p\phi(EU_1(2)) + (1-p)\phi(EU_2(2))\right).$$

From these expressions, $V_{recursive} > V_{reduced}$ iff

$$\mathbf{E}_i\phi(EU_i(1) + \beta X) > \mathbf{E}_i\phi(EU_i(1) + \beta EU_i(2)),$$

i.e., we are asking whether adding a certainty equivalent X of the risk $EU_i(2)$ to the risk $EU_i(1)$ is preferred to adding the risk $EU_i(2)$ itself. There is no general answer to this question – it depends on the values of the $EU_i(t)$, and the specific functional form of ϕ. Moreover, since the $EU_i(t)$ are all implicitly complicated functions of η, and are likely non-monotonic, and discontinuous at $\eta = 1$, we can expect to get ranking reversals as η changes and affects the values of the $EU_i(t)$ non-uniformly.

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